Pearson Edexcel

Mark Scheme (Results)

Autumn 2020

Pearson Edexcel GCE Further Mathematics
AS Further Decision 2 Paper 8FM0_28

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

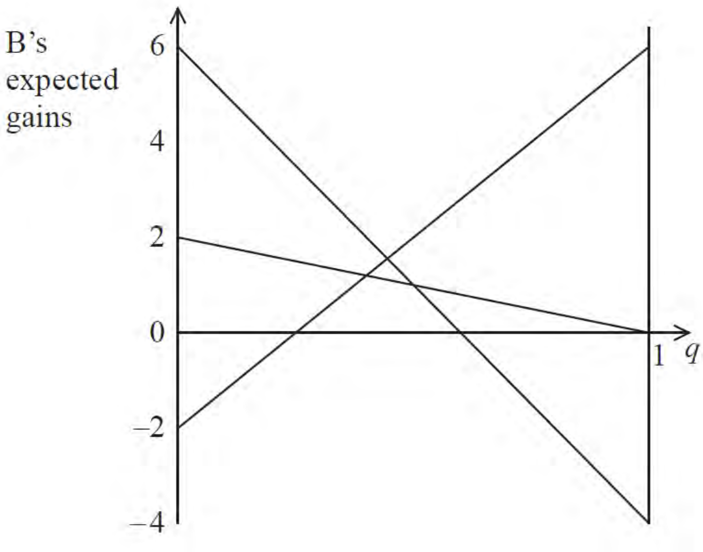
Question	Scheme	Marks	AOs
1(a)	**(i)** (x =) 9	B1	1.1b
	(ii) (y =) 14	B1	1.1b
		(2)	
(b)	SA, FE, FT	B1	1.1b
		(1)	
(c)	**(i)** Value of cut C_1 = 18 + 12 + 17 + 26 = 73	B1	1.1b
	(ii) Value of cut C_2 = 18 + 37 + 17 + 26 = 98	B1	1.1b
		(2)	
(d)	e.g. SCFBET, SBCFBET	B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem Identification of cut through SA, AB, BE, FE and FT Value of flow = 57 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
			(9 marks)

Notes

(a)(i)
B1: Cao
(ii)
B1: Cao
(b)
B1: Cao
(c)(i)
B1: Cao
(ii)
B1: Cao
(d)
B1: A correct flow-augmenting route
(e)
M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through saturated arcs) – if the cut is only given in terms of the capacity of the arcs (rather than in terms of the nodes at each end) then M1 only in this part
A1: Use appropriate process of finding a minimum cut – cut and value correct
A1: Correct deduction that the flow is maximal

Question	Scheme	Marks	AOs
2(a)	Subtract each entry from a constant (e.g. 98) to convert from maximisation problem to minimisation	B1	1.1a
	Add an appropriate large value to cell CQ (e.g. twice the largest value) to make CQ unattractive	B1	3.5c
		(2)	
(b)	e.g. $\begin{array}{c\|cccc} & P & Q & R & S \\ \hline A & 26 & 0 & 39 & 14 \\ B & 31 & 11 & 30 & 12 \\ C & 28 & 78 & 36 & 19 \\ D & 20 & 5 & 34 & 17 \end{array}$	B1	1.1b
		(1)	
(c)	No reduction for row A, reduce row B by 11, reduce row C by 19 and row D by 5 (or equivalent). Reduce column P by 9 and column R by 17, no reduction in columns Q and S.	B1	2.4
	Reducing rows and columns gives $\begin{array}{c\|cccc} & P & Q & R & S \\ \hline A & 26 & 0 & 39 & 14 \\ B & 20 & 0 & 19 & 1 \\ C & 9 & 59 & 17 & 0 \\ D & 15 & 0 & 29 & 12 \end{array}$ followed by $\begin{array}{c\|cccc} & P & Q & R & S \\ \hline A & 17 & 0 & 22 & 14 \\ B & 11 & 0 & 2 & 1 \\ C & 0 & 59 & 0 & 0 \\ D & 6 & 0 & 12 & 12 \end{array}$	M1	1.1b
	Two lines required to cover the zeros hence solution is not optimal (augment by 1) $\begin{array}{c\|cccc} & P & Q & R & S \\ \hline A & 16 & 0 & 21 & 13 \\ B & 10 & 0 & 1 & 0 \\ C & 0 & 60 & 0 & 0 \\ D & 5 & 0 & 11 & 11 \end{array}$	M1	1.1b
	Three lines required to cover the zeros hence solution is not optimal (augment by 5) e.g. $\begin{array}{c\|cccc} & P & Q & R & S \\ \hline A & 11 & 0 & 16 & 8 \\ B & 10 & 5 & 1 & 0 \\ C & 0 & 65 & 0 & 0 \\ D & 0 & 0 & 6 & 6 \end{array}$	M1	1.1b
	Four lines required to cover the zeros hence solution is optimal	B1	2.4
	A – Q, B – S, C – R, D – P	A1	2.2a
		(6)	
		(9 marks)	

Notes
(a) **B1**: Valid statement regarding converting a maximisation problem to a minimisation problem **B1**: Explain the need to add an unattractive value to cell CQ (note that candidates may first assign a negative value to the CQ entry and then subtract) **(b)** **B1:** Mark awarded when both steps complete (subtraction and addition of extra cell) **(c)** **B1:** Correct statements regarding row and column reduction **M1:** Simplifying the initial matrix by reducing rows and then columns **M1:** Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 2 lines needed to 3 lines needed **M1:** Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table) **B1:** Correct statements regarding the minimum number of lines to cover zeros **A1:** CSO on final table (so must have scored all previous M (but not necessarily the B) marks in this part) + deduction of the correct allocation

Question	Scheme	Marks	AOs
3(a)	**(i)** 7	B1	3.4
	(ii) 6	B1	3.4
		(2)	
(b)	**(i)** Row minima: –6, –2, –6 max is –2 Column maxima: 4, 2, 6 min is 2	M1 A1	1.1b 1.1b
	Play-safe for Team A is Noel and play-safe for Team B is Qaasim	A1	1.1b
	(ii) Row(maximin) $\neq$ Col(minimax) therefore game is not stable	B1	2.4
		(4)	
(c)	e.g. If Team A plays safe then Team B should also play their play-safe option which is Qaasim as by playing Qaasim they will gain 2 compared to gaining zero (if playing Paul) or losing 6 (if playing Rashid)	B1	2.4
		(1)	
(d)	Let B play Paul with probability q and Qaasim with probability $1 – q$	B1	3.3
	If A plays Mischa, B's gains are $-\big(4q+(-6)(1-q)\big)=6-10q$ If A plays Noel, B's gains are $-\big(-2(1-q)\big)=2-2q$ If A plays Olive, B's gains are $-\big(-6q+2(1-q)\big)=-2+8q$	M1 A1	1.1b 1.1b
		M1 A1	1.1b 1.1b
	$2 – 2q = – 2 + 8q \Rightarrow q = 2/5$	A1	1.1b
	Team B should play Paul with probability 0.4 and play Qaasim with probability 0.6	A1ft	3.2a
		(7)	
			(14 marks)

Notes
(a)(i) **B1:** cao **(a)(ii)** **B1:** cao **(b)(i)** **M1:** finding row minimums and column maximums – condone one error **A1:** row minima and column maxima correct **A1:** correct play safes for both teams **(b)(ii)** **B1:** row maximin (-2) $\neq$ col minimax (2) so not stable **(c)** **B1:** cao (or equivalent – e.g. Qaasim because -2 is the lowest value in Noel's row) – explanation must involve consideration of values and not just (for example) a general statement that Qaasim will gain the most **(d)** **B1:** defining variable q **M1:** setting up three expressions in terms of q **A1:** all three expressions correct – allow correct un-simplified expressions for this mark **M1:** axes correct, at least one line correctly drawn for their expressions **A1:** correct graph **A1:** using the graph to obtain the correct probability expressions leading to the correct value of q **A1ft:** interpret their value of q in the context of the question – must refer to play/choose and the two players Note that in **(d)** candidates may use p (or another letter) instead of q which is fine for full marks. Also, the three expressions may be the negative of what is giving in the main scheme (e.g. $10q – 6$, $2q – 2$ and $–8q + 2$) and this is fine for the first 5 marks in **(d)**. For the final two marks though they would need to consider the optimal point reading from the top (rather than the bottom) of their graph. No follow through for the final mark if they do not read off **their** graph correctly.

Question	Scheme	Marks	AOs
4	(aux equation $2m-1=0\Rightarrow$) complementary function is $A\left(\frac{1}{2}\right)^n$	B1	2.1
	Particular solution try $u_n=\lambda n^2+\beta n+\alpha$ and substitute into recurrence relation	M1	1.1b
	$2\lambda n^2+2\beta n+2\alpha=(\lambda-k)n^2+(-2\lambda+\beta)n+(\lambda-\beta+\alpha)$ $\Rightarrow 2\lambda=\lambda-k$ $2\beta=-2\lambda+\beta$ $2\alpha=\lambda-\beta+\alpha$	M1	1.1b
	$u_n=A\left(\frac{1}{2}\right)^n-kn^2+2kn-3k$	A1	1.1b
	$u_0=A-3k, u_2=\frac{1}{4}A-3k \Rightarrow 4\left(\frac{1}{4}A-3k\right)-(A-3k)=27k^2$	M1	3.1a
	$27k^2+9k=0\Rightarrow k=-\frac{1}{3}\quad(k\neq 0)$	A1ft	1.1b
	As n becomes large $A\left(\frac{1}{2}\right)^n\to 0$	B1	2.4
	$u_n\to\frac{1}{3}n^2-\frac{2}{3}n+1\quad\left(a=\frac{1}{3},b=-\frac{2}{3},c=1\right)$	A1	2.2a
		(8 marks)	

Notes

B1: cao
M1: correct form for the particular solution and substituted into recurrence relation
M1: compares coefficients and setting up all three equations in λ,β,α
A1: correct general solution (or with consistent value of k)
M1: use initial condition to obtain a quadratic equation in k
A1ft: correct solution for k following through their general solution
B1: correct explanation that the exponential term tends to zero as n becomes large
A1: cao

Alternative for third M mark: Note that candidates may calculate k immediately by eliminating u_1 from $2u_1=u_0-k$ and $2u_2=u_1-4k$ and comparing with $4u_2-u_0=27k^2$ to obtain a quadratic in k